Wind Energy Fundamentals

Great Website

http://drømstørre.dk/wp-content/wind/miller/windpower%20web/en/tour/wres/shelves.htm

Lecture - Learning Objectives

At the end of this lecture you should be able to:

- Understand the thermodynamic limit for wind energy
- Understand how thrust occurs and how it turns the blade
- Understand the interworkings of a wind turbine
- Understand magnets and how turning of the blade is converted to electrical energy.

Power

- How much power does wind provide.

- Lets break this down as followed:

$$E = {}^{1}/_{2}\, mv^2$$ ← Eqn for kinetic energy

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta {}^{1}/_{2} mv^2}{\Delta t} = {}^{1}/_{2}\, \dot{m}v^2 = {}^{1}/_{2}\,(A\rho v)v^2$$

↑ Mass flow rate

$$P_{wind} = {}^{1}/_{2}\, A\rho v^3$$

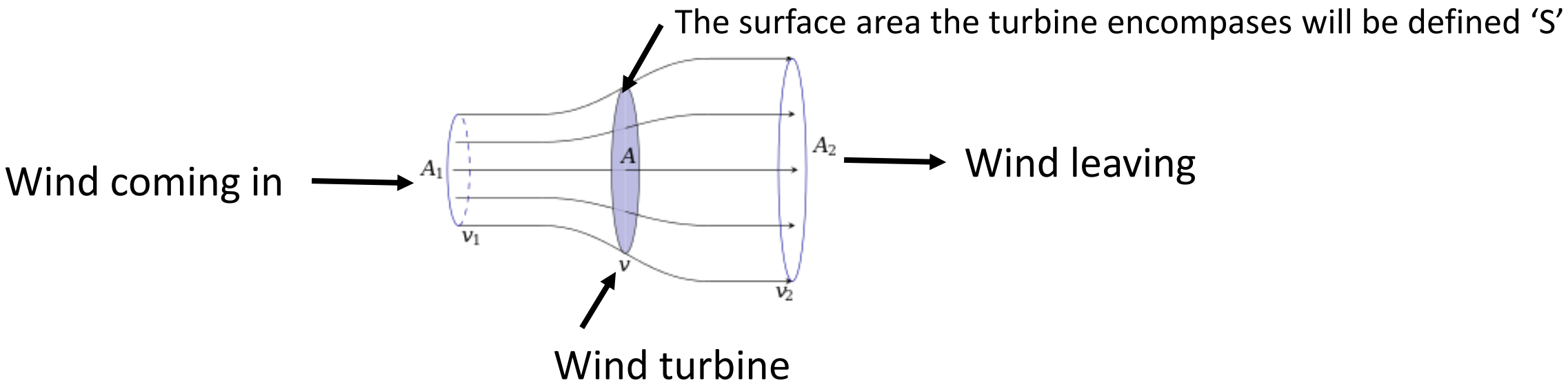
E = Energy
m = mass
v = velocity
P = power
T = time
S = area

- Power scales with velocity to the 3rd power. (Practical losses often scale with velocity to the 1st power, thus often actual power scales with velocty to the 2nd power)

- Now that we know how much power wind provides is there a limit to how much of this we can obtain? (hint: The answer is yes)

Power from Wind Turbines

- The total energy from the wind assumes the wind molecules come to a complete stop.
- We need to get rid of the stopped wind molecules, thus we need some velocity to remove them.
- This means that the wind after the turbine (A_2, V_2) will have a smaller, yet significant wind velocity than the incoming wind (A_1, V_1) .



Wind Power from Turbines

- From kinetic energy we know :

$$E = {}^{1}/_{2}\, mv^2$$

Different wind velocity before and after wind turbine

$$\Delta E = {}^{1}/_{2}\, m(v_1^2 - v_2^2)$$

Mass flow rate

$$P = \frac{\Delta E}{\Delta t} = {}^{1}/_{2}\, \frac{m}{\Delta t}(v_1^2 - v_2^2)$$

- In this approach we determined the power from an energy balance.

- We can also calculate the power using a force balance

Wind Power via Newton's 2nd Law

- If we denote force from wind as:

Mass flow rate

$$F = ma = m * \frac{dv}{dt} = \frac{m}{\Delta t}\Delta v = \dot{m}\Delta v = \rho S v_{@A}(v_1 - v_2)$$

The change in energy occurs at the turbine blade (i.e. @a)

$$P = \frac{dE}{dt} = \frac{d(F*x)}{dt} = F * \frac{dx}{dt} = \rho S v_{@A}(v_1 - v_2)*v_{@A} = \rho S v_{@A}^2(v_1 - v_2)$$

- This is strange. Using a kinetic energy approach and a force balance approach both gives us power equations.

Energy Balance vs Force Balance

- From kinetic energy we know our turbine power is: $P = {}^{1}/_{2}\,\rho S v_{@A}(v_1^2 - v_2^2)$
- From force balance we know our turbine power is: $P = \rho S v_{@A}^2 (v_1 - v_2)$
- What this tells us is that v_1, v_2 and $v_{@A}$ are inter-related. If we set 2 of these values the third one will be automatically set.
- If we set our energy balance equation equal to our force balance and solve for $v_{@A}$ we get:

$$ {}^{1}/_{2}\,\rho S v_{@A}(v_1^2 - v_2^2) = \rho S v_{@A}^2 (v_1 - v_2) $$

Boring algebra

$$ v_{@A} = {}^{1}/_{2}\,(v_2 + v_1) $$

Maximum Wind Power

- From kinetic energy we know : $P = {}^{1}/_{2}\,\rho S v_{@A}(v_1^2 - v_2^2)$

or

$$P = {}^{1}/_{4}\,\rho S(v_2 + v_1)(v_1^2 - v_2^2)$$

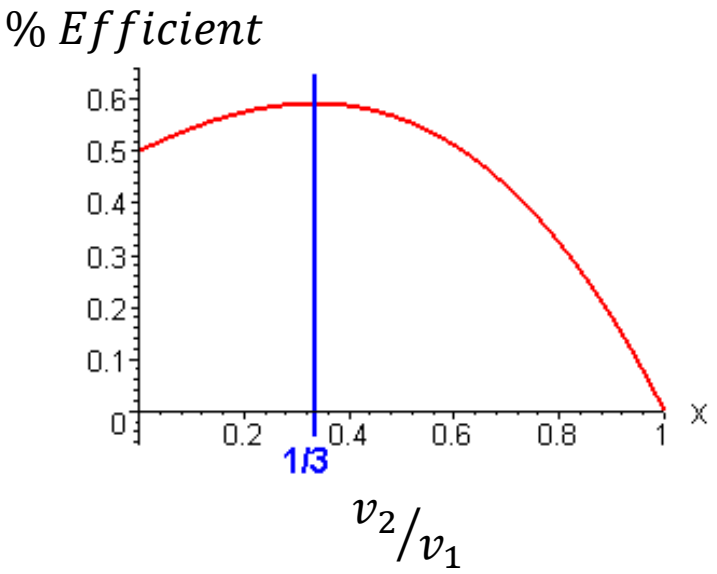
  - Now we know what $v_{@A}$ is, however we have to figure out what our maximum power can be.
  - The way to find maximum power is to take the derivative and set it to 0.
  - But what should we take the derivative of? The variable in this system that we can control is v_2, so you may think take the derivative with respect to v_2.
  - However what we really are interested in is the ratio of v_2 to v_1 and thus we should take our derivative with respect to that.
  - First we need to get P in terms of v_2/v_1. After a bit of algebra we get:

$$P = {}^{1}/_{4}\,\rho S v_1^3\left(-\left(\frac{v_2}{v_1}\right)^3 - \left(\frac{v_2}{v_1}\right)^2 + \frac{v_2}{v_1} + 1\right)$$

Maximum Wind Power

- We need to take the derivative of : $P = {}^{1}/_{4}\,\rho S v_1^3\left(-\left(\frac{v_2}{v_1}\right)^3 - \left(\frac{v_2}{v_1}\right)^2 + \frac{v_2}{v_1} + 1\right)$ and set it equal to zero

- Thus $\dfrac{dP}{d\left(\frac{v_2}{v_1}\right)} = 0$ $\Longrightarrow$ $\dfrac{v_2}{v_1} = {}^{1}/_{3}$



- And thus the power from our wind turbine is:

$$P = {}^{1}/_{4}\,\rho S v_1^3\left(-\left(\frac{v_2}{v_1}\right)^3 - \left(\frac{v_2}{v_1}\right)^2 + \frac{v_2}{v_1} + 1\right) = {}^{16}/_{27} \times {}^{1}/_{2}\,\rho S v_1^3$$

Another way to look at things

- If the maximum wind turbine power is : $P_{Turbine} = {}^{16}/_{27} \times {}^{1}/_{2}\, \rho S v_1^3$
- And the total power from the incoming wind is: $P_{Wind} = {}^{1}/_{2}\, m v_1^2$ (m=Mass flow rate)

$$P_{Wind} = {}^{1}/_{2}\, \rho S v_1^3$$

- Thus : $$\frac{P_{Turbine}}{P_{Wind}} = \frac{{}^{16}/_{27} \times {}^{1}/_{2}\, \rho S v_1^3}{{}^{1}/_{2}\, \rho S v_1^3} = {}^{16}/_{27} = 59\%$$

- Thus the maximum efficiency is 16/27 (or 59%), which is called the Betz Limit.

- A major key to achieving this is to make sure you have the proper v_2/v_1 ratio

Maximum Wind Power

Instead of this diagram, we can use this diagram

Figure A

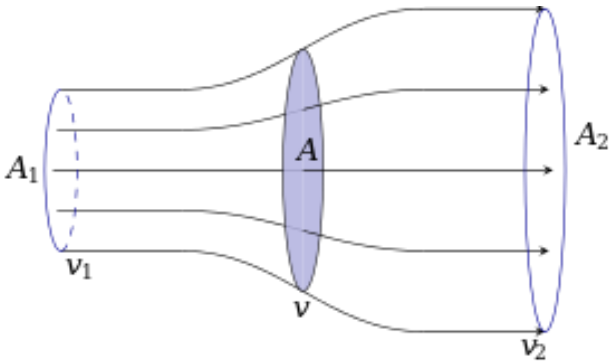
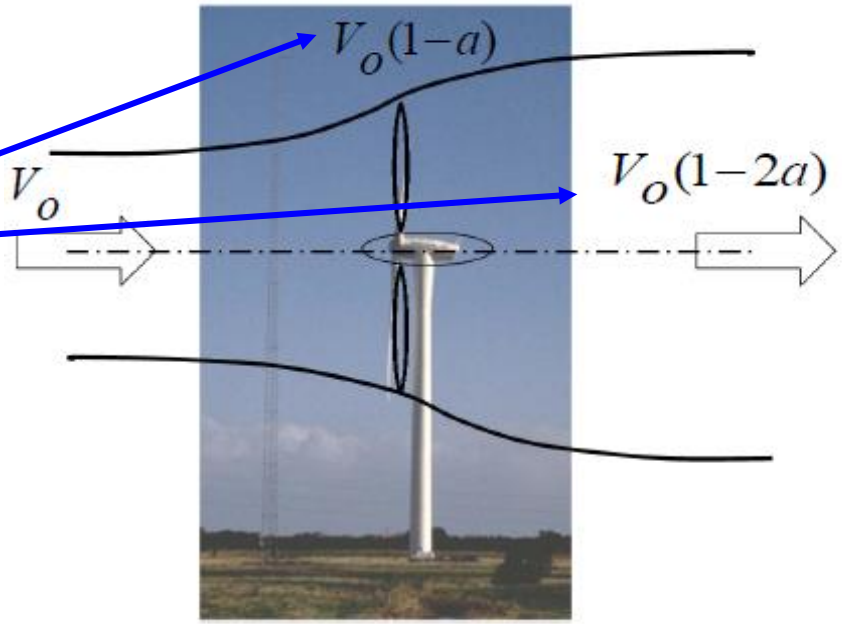


Figure B



These ratios are true because of what we derived earlier:

$$v_{@A} = {}^{1}/_{2}\,(v_2 + v_1)$$

Axial interference cofactor $\longrightarrow$ $a = 1 - \dfrac{v}{V_0}$ $\longleftarrow$ v= velocity at wind turbine

$$a_{max} = {}^{1}/_{3}$$

Coefficient of performance

- Our coefficient of performance (C_p) is simply:

$$C_p = \frac{P_{Turbine}}{P_{Wind}}$$

Terminology from Figure A of previous slide

$$= \frac{\rho S v_{@A}^2 (v_2 - v_1)}{{}^1/_2\, \rho S V_1^3}$$

Terminology from Figure B of previous slide

$$= \frac{\rho S v^2 (V_0(1-2a) - V_0)}{{}^1/_2\, \rho S V_0^3}$$

Boring Algebra

$$C_p = 4a(1-a)^2$$

- If a=1/3 C_p will be 59%, but any other a will yield a different C_p.

Thrust

- Given our velocity slows down across the wind turbine we lose some momentum.
- Conservation of momentum says this must go somewhere.
- This momentum goes into the thrust of our wind blades, which then provides the basis for energy in our turbines
- Thrust (T) is then:

Using notation from Figure B on Slide 12

$$T = \frac{dm}{dt} v_{Blade} = \rho S v V_0 2a = 2\rho S v V_0 \left(1 - \frac{v}{V_0}\right) = 2\rho S v (V_0 - v)$$

- Power can also be related in terms of thrust:

Our thrust is our force

$$P = \frac{E}{t} = \frac{F \times d}{t} = \mathrm{F} \times \frac{d}{t} = Fv = Tv = 2\rho S v^2 (V_0 - v)$$

Thrust

- Since we have a C_p, we can also have a thrust coefficient (C_T) as followed:

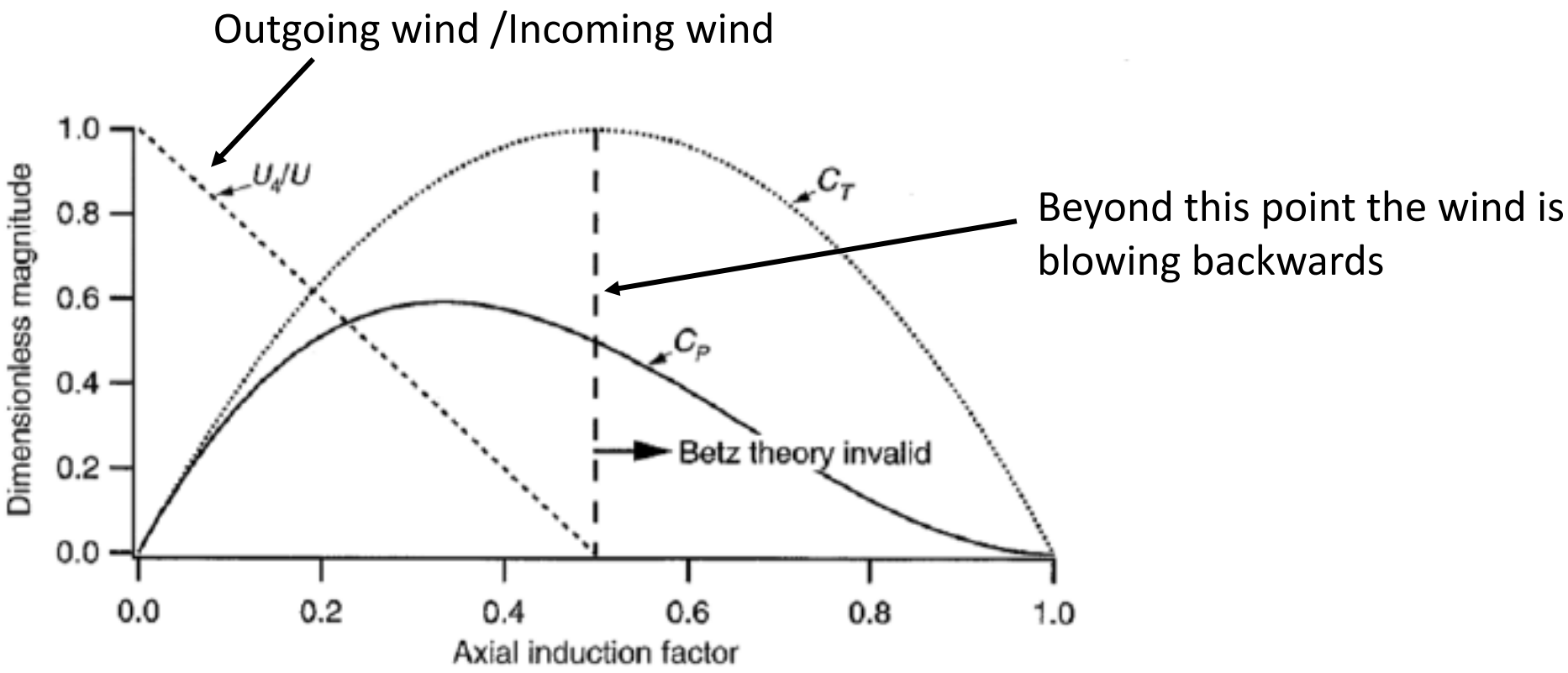
$$C_T = \frac{Thrust\ Force}{Maximum\ Force\ from\ wind} = \frac{2\rho S v(V_0 - v)}{{}^{1}/_{2}\,\rho A v^2}$$

- If C_T is near 1 that means there is incredible force on our wind turbine, but the velocity of the air is very small. (In terms of solar cells: think lots of voltage very little current)

- If C_T is near 0 that means there is little force on our wind turbine, but the velocity of the air is large. (In terms of solar cells: think lots of current very little voltage)

Optimal Thrust Coefficient

- The optimal C_T is 8/9.
- The table below plots C_P and C_T as a function of *a*

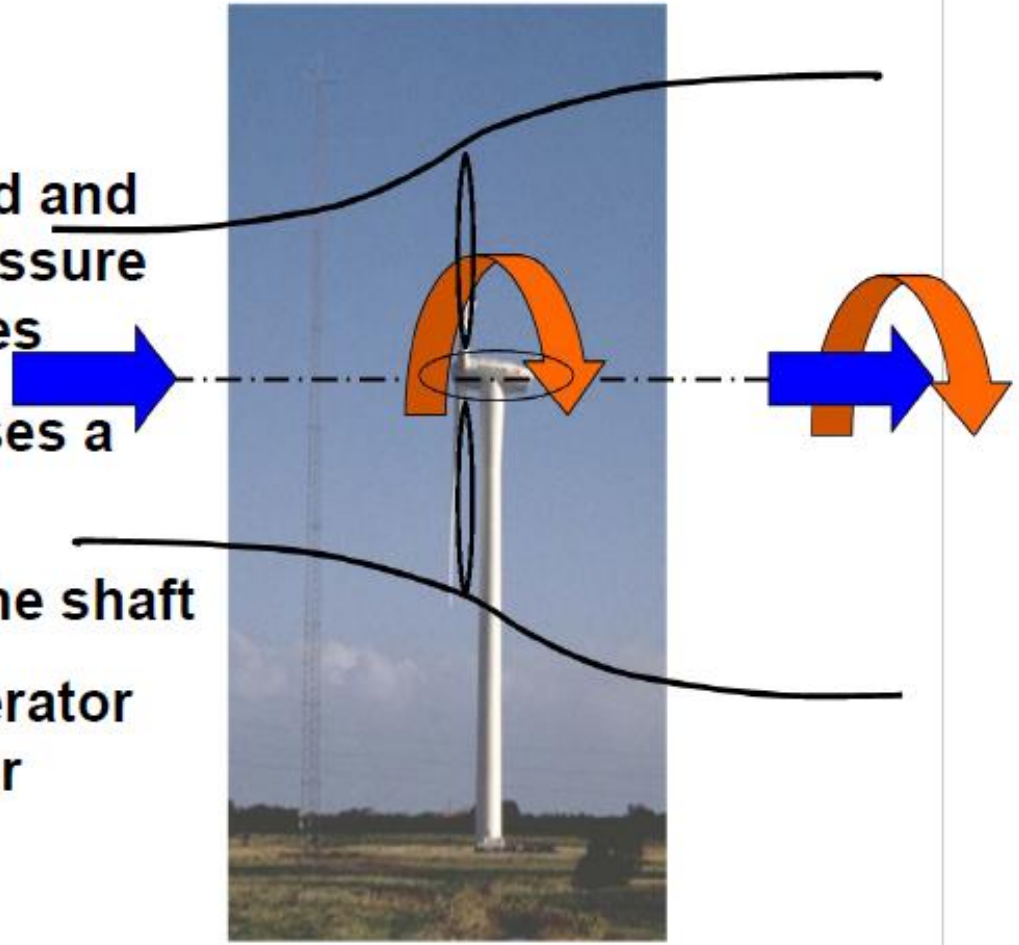


'Wind Energy Explained' pg 88

Wind Energy

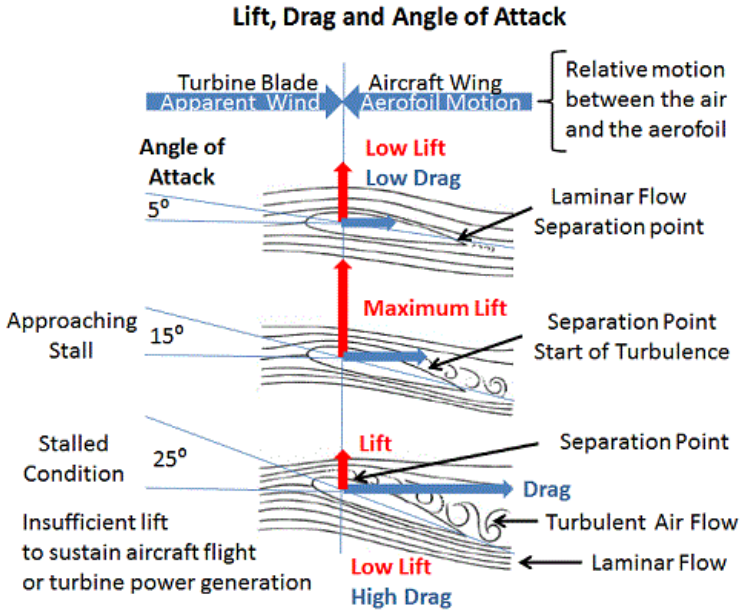
How does a wind turbine work?

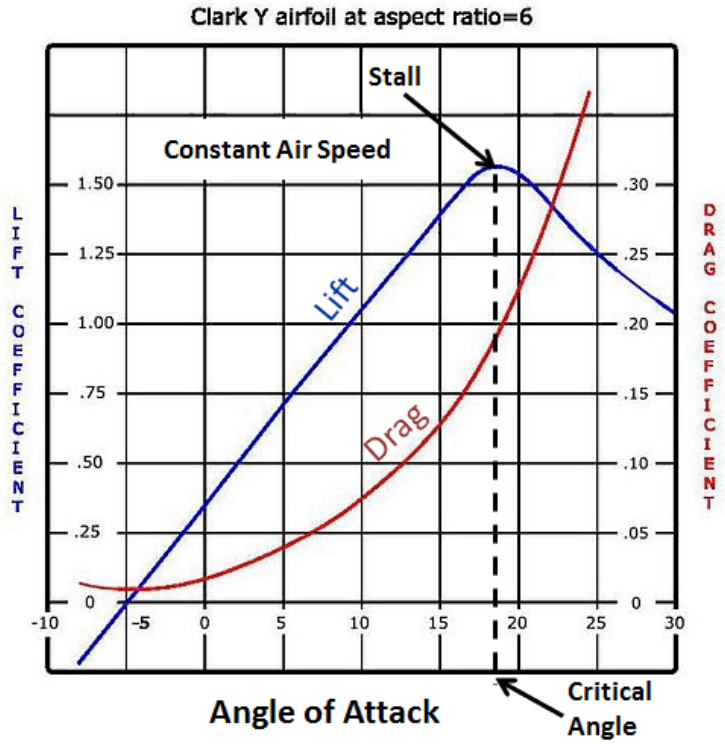
- The wind hits the rotor plane
- The combinition of wind speed and blade rotation results in a pressure distribution on the rotor blades
- The pressure distribution causes a turning moment (torque)
- The turning moment rotates the shaft
- The shaft is coupled to a generator that produces electrical power



How do we actually move the blade ?

- The blade is designed so it forces the wind to have a lower pressure on one side that so it gives the blade 'Lift'
- The key is mitigating turbulence, because this creates 'Drag'.
- Varying the angle changes the Lift and Drag ratio.
- This is a function of wind blade design
- Beyond a certain angle the turbulence reaches the lift point, and drastically decreases lift.
  - This is called 'Stall'
- The exact aerodynmics is beyond the scope of this class.





Looking at Angular Velocity

- We can use Bernoulli's Equation to look at force balance at the beginning (B) and end (A) of the wind blade

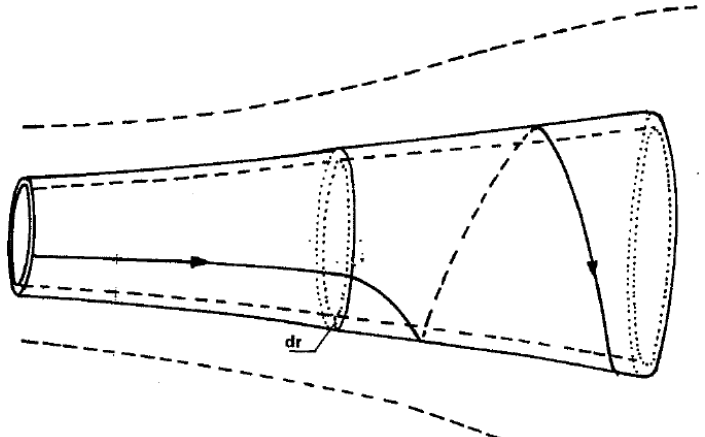
The wind after the turbine has more angular velocity than before

$$P_B + {}^1/_2\,\rho\Omega^2 r^2 = P_A + {}^1/_2\,\rho(\Omega + \omega)^2 r^2$$

Boring Algebra

$$P_B - P_A = \rho\omega r^2\left(\Omega + {}^1/_2\,\omega\right)$$

Ω = angular velocity
ω = incremental increase in angular velocity
r = distance along the blade
R = total blade radius



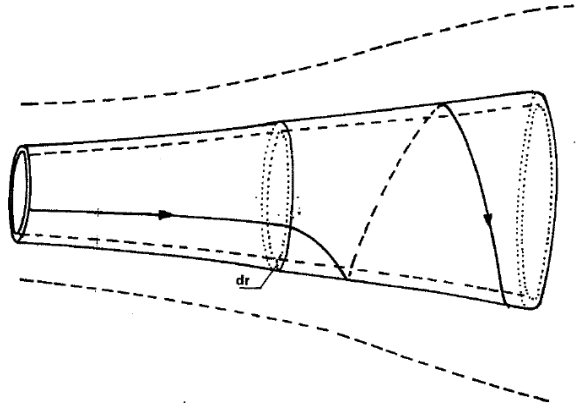
Thrust via Bernoulli's equation

- Thrust on a minute element (dT) gives us:

Force term

Incremental area across circle the turbine blade encompasses

$$dT = \overbrace{(P_B - P_A)dA} = \rho\omega r^2\left(\Omega + {}^{1}/_{2}\,\omega\right)\overbrace{2\pi r dr}$$



- Lets create an angular induction factor as followed

$$a' = \frac{\omega}{2\Omega}$$

- Applying this to our thrust equation yields:

Boring Algebra

$$dT = 4a'(1 + a')\rho\Omega r^2 \pi r dr$$

Revisiting Our Linear Thrust

- Originally we denoted thrust as:

$$T = 2\rho S v(V_0 - v)$$

- If instead of using the whole surface area (S) we focus on a differential ring for a differential thrust. Thus we get:

$$\mathrm{d}T = 2\rho(2\pi r dr)v(V_0 - v)$$

Remember

$$a = 1 - \frac{v}{V_0}$$

- Now we want to get this in terms of *a* as followed:

$$T = 2\rho(2\pi r dr)v(V_0 - v)$$

Boring Algebra

$$T = 2\rho a(1 - a){V_0}^2\,(2\pi r dr)$$

We Calculated Thrust 2 Different ways

- Linear momentum gave us: $dT = 2\rho a(1-a){V_0}^2\,(2\pi r dr)$
- Bernoulli's equation gave us: $dT = 4a'(1+a')\rho\Omega r^2 \pi r dr$
- Setting the 2 thrusts equal to each other gives us a relationship between angular turbine velocity and linear wind velocity:

Local speed ratio

$$\frac{a(1-a)}{a'(1+a')} = \frac{\Omega^2 r^2}{{V_0}^2} = \lambda_r^2$$

Remember

$$a' = \frac{\omega}{2\Omega}$$

$$a = 1 - \frac{v}{V_0}$$

- We call λ_r the local speed ratio: $\lambda_r = \dfrac{\Omega r}{V_0}$
- Whereas λ is the tip speed ratio: $\lambda = \dfrac{\Omega R}{V_0}$

Torque

- To push the turbine blade around we must have a given torque.

- This torque comes from change in angular momentum of wake (conservation of angular momentum).
- If we look at a differential element of Torque on our wind blade we have:

$$dQ = d(\dot{m}(\omega r) \times r)$$

- We can get this in terms of our incoming velocity since from Slide 11:

$$v = V_0(1 - a)$$

- We can get eliminate 'ω' by noting that from Slide 19:

$$a' = \frac{\omega}{2\Omega}$$

$$dQ = V_0(1 - a)2\Omega a' \rho 2\pi r dr$$

Torque

- We can determine our differential <u>power</u> as a function of torque as followed:

$$dP = d\left(\frac{F \times d}{t}\right) = dQ \times \Omega$$

$$dP = 2a'(1-a)\rho v 2\pi r \Omega^2 dr$$

P = Power
d = distance
T = time
v = velocity

- If we want this in terms of our tip speed ratio we will get:

Boring Algebra

$$dP = 4\rho A v^3 \frac{1}{\lambda^2} a'(1-a)\lambda_r^3 d\lambda$$

$$C_P = \int \frac{dP}{P_{Wind}} = \int \frac{dP}{{}^1/_2 A\rho V_0^3} = \frac{8}{\lambda^2}\int_0^{\lambda} a'(1-a)\lambda_r^3 d\lambda$$

<u>Remember</u>

$$\lambda_r = \frac{\Omega r}{V_0}$$

$$\lambda = \frac{\Omega R}{V_0}$$

$$a' = \frac{\omega}{2\Omega}$$

$$a = 1 - \frac{v}{V_0}$$

What is ω?

- We have *a'* which is a function of ω, but what is this?
- Without derivation we can relate *a'* to a and λ_r as followed

$$a' = -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{\lambda_r^2}a(1-a)}$$

- In terms of ω this is:

$$\omega = 2\Omega\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{\lambda_r^2}a(1-a)}\right)$$

P = Power
d = distance
T = time
v = velocity

Remember

$$\lambda_r = \frac{\Omega r}{V_0}$$

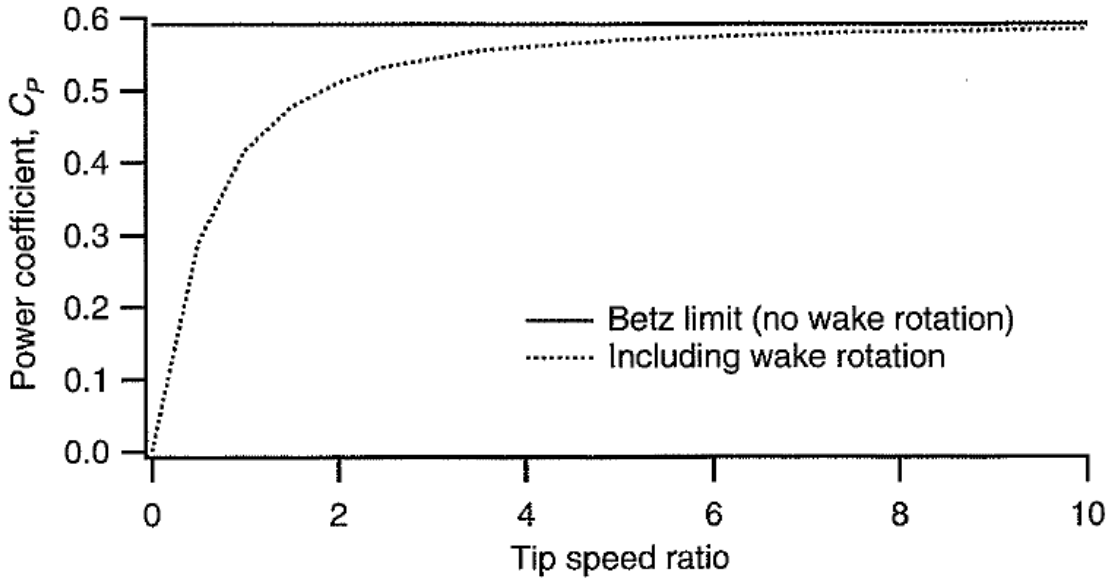
$$\lambda = \frac{\Omega R}{V_0}$$

$$a' = \frac{\omega}{2\Omega}$$

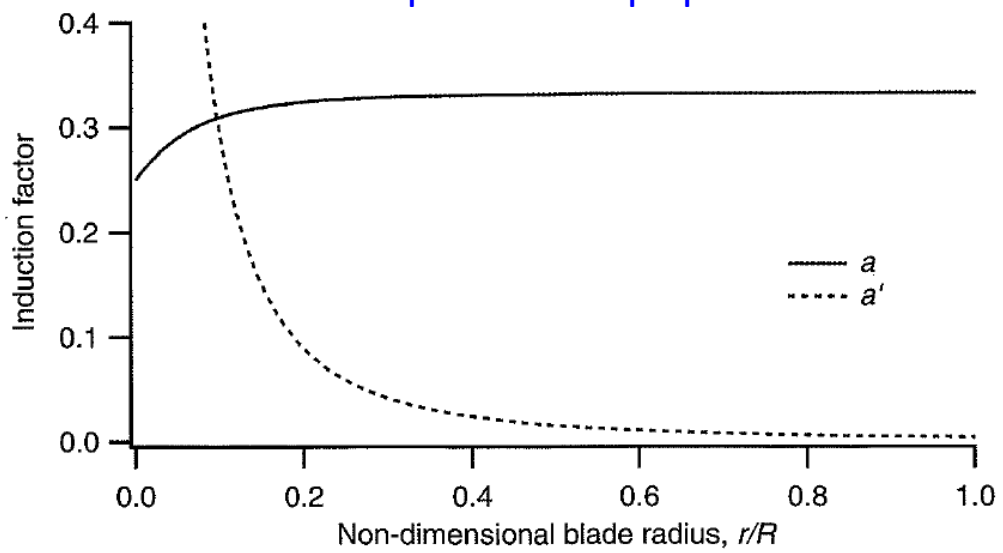
$$a = 1 - \frac{v}{V_0}$$

Understanding Wake Rotation

- We know C_p is a function of λ, but what does this mean?
- If your λ is too slow compared to the wind speed you lose efficiency.
- The second graph shows if λ is small, the 'a' value near the middle of the wind turbine is lower than the 1/3 that it should be.

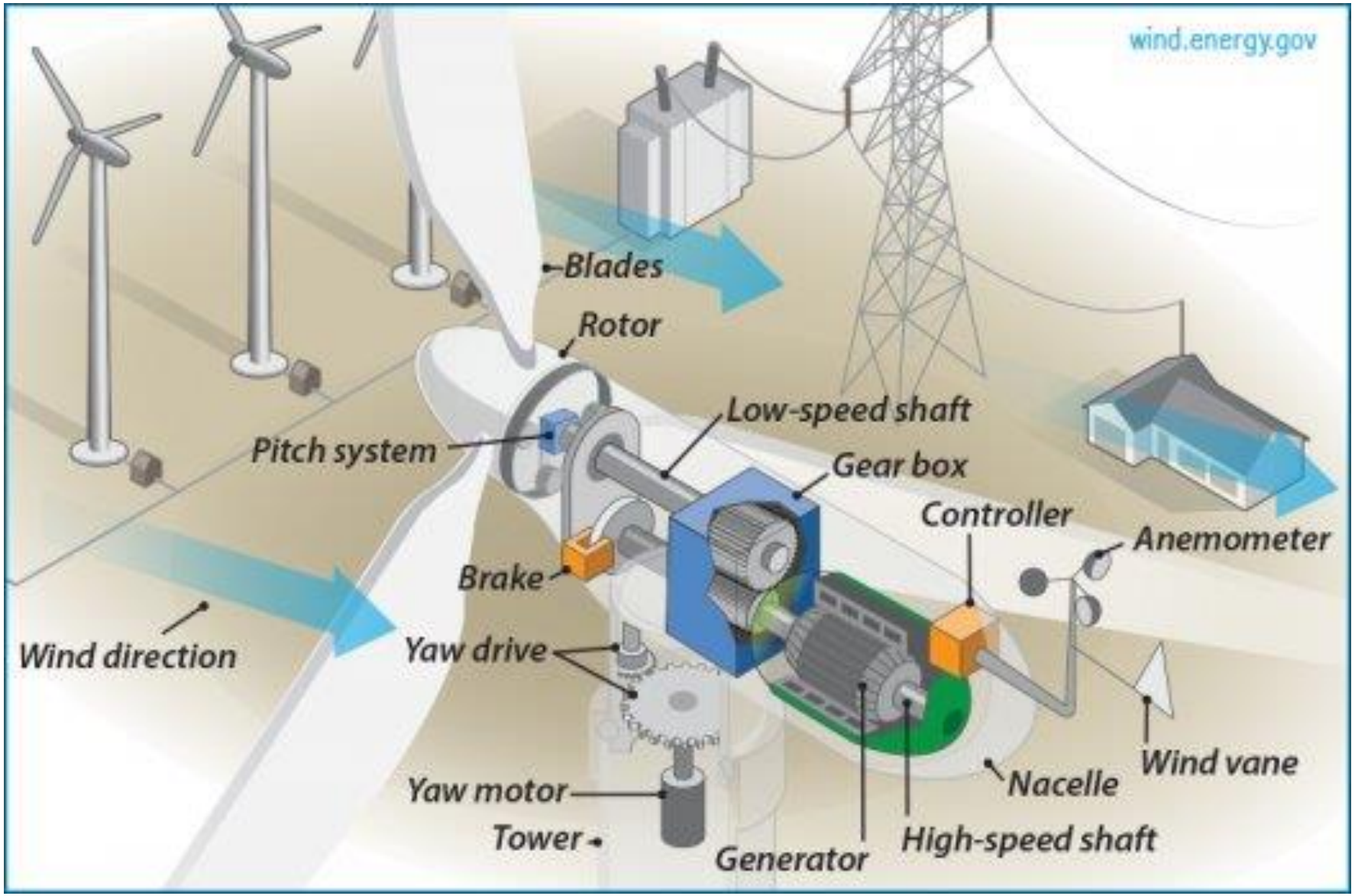


Graph uses a tip speed ratio of 7.5



BREAK

Inside Wind Turbines



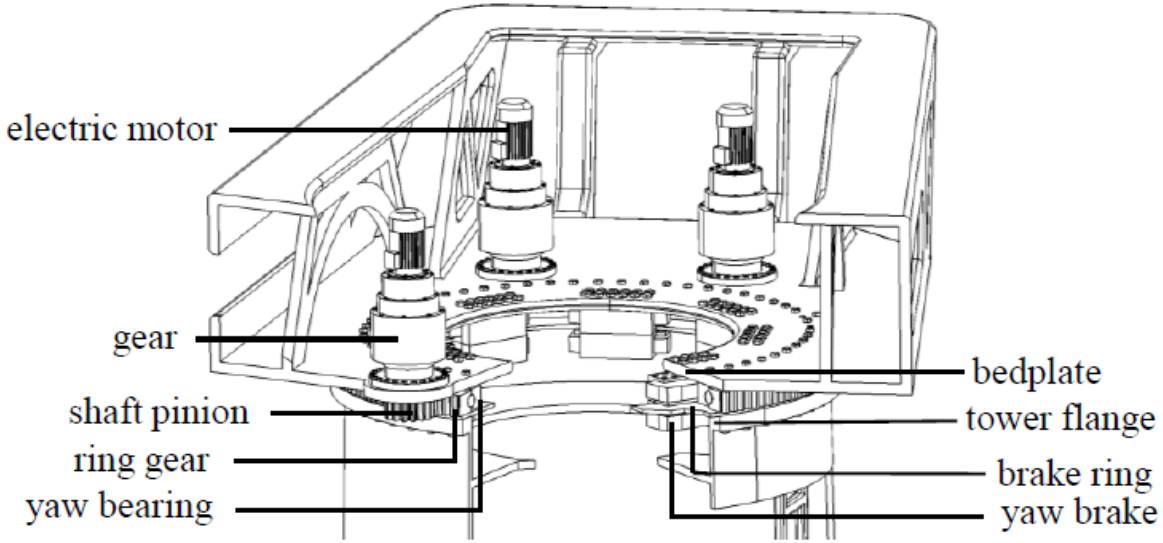
Yaw System

- The Yaw system rotates the blades in the direction of the wind.
- The wind vane is responsible for communicating and controlling rotation.
- Remember Inertia & Torque

$$I = \frac{M}{V}\int_0^r r^2{}_\perp \, dV \qquad \& \qquad \tau = I\alpha$$

τ = torque
α = angular acceleration
r = radius perpendicular to axis
V = volume

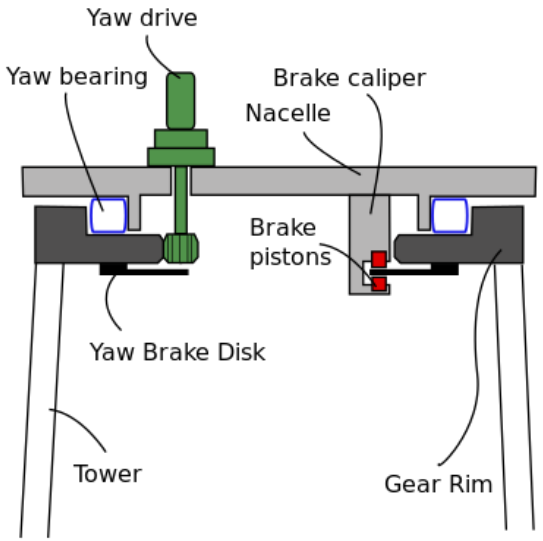
- Heavy magnets, means high Torque is needed to move/accelerate turbine
- Yaw motor Torque = 200,000 N*m
  Car motor Torque = 250 N*m
- This is 1-5% of total cost.

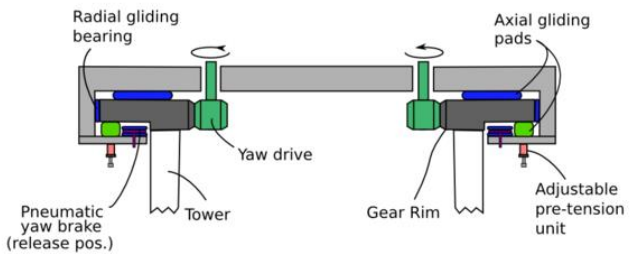


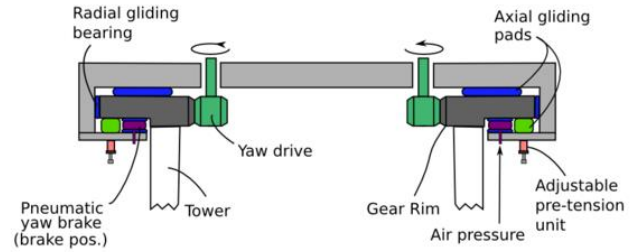
M-G Kim and Dalhoff, P.H., (2014) J. Phys,: Con. Ser. 524 012086
Doi: 10.1088/1742-6596/524/1/012086

Yaw System

- Typically regular ball bearings are used
  - Double row bearing are used sometimes to decrease wear, but with higher costs
- Sliding bearings
  - Advantages: Sliding can help in braking
  - Disadvantages: Inconsistent sliding, can be loud
- Yaw Braking
  - This is typically done using hydraulics
  - Hydraulics leak, needing maintenance and can even cause fires.
- In general yaw systems and non-technically optimized to reduce on costs.

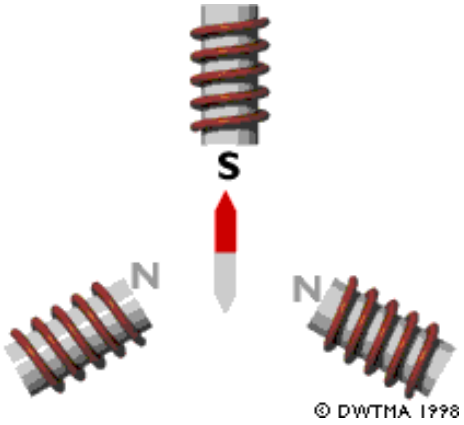




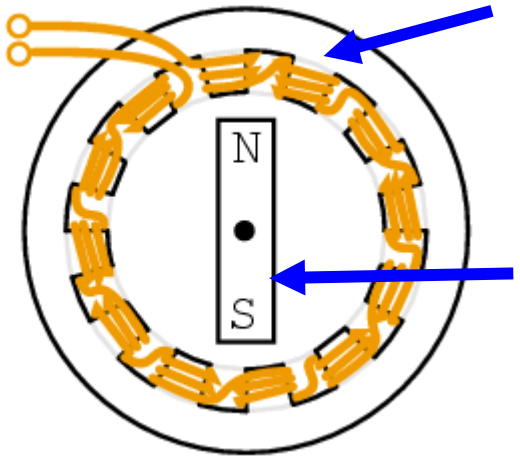


Generators

- Typical 690 V three-phase alternating current (AC-current)
- Thus gearing is essential to produce appropriate current
- A permanent magnet is used with copper windings to generate charge
- Need cooling-normally air, sometimes water



3 phase motor

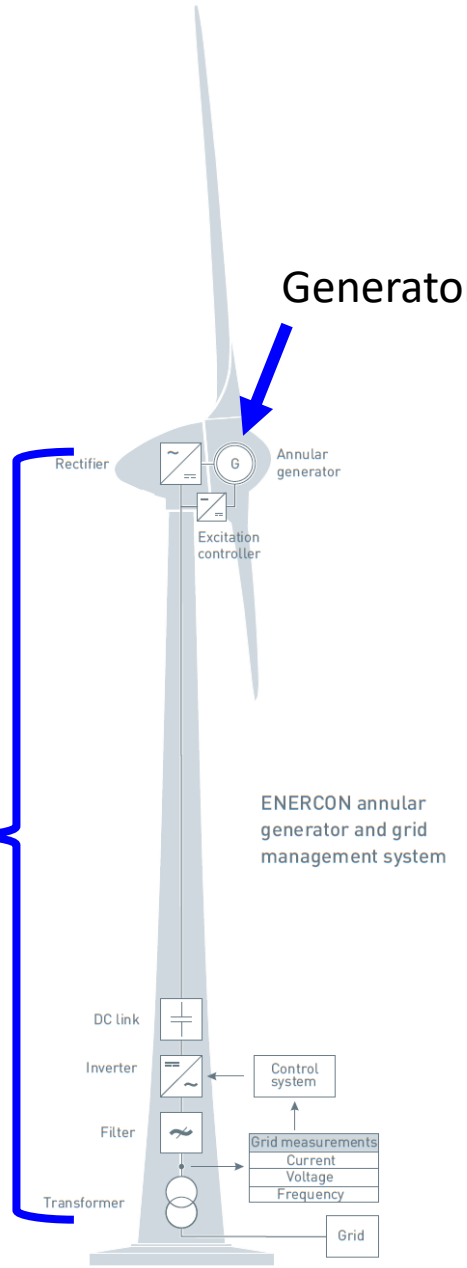


Stator – This is what is static. Normally this is the copper wires

Rotor – This is what rotates. This is normally the permanent magnet

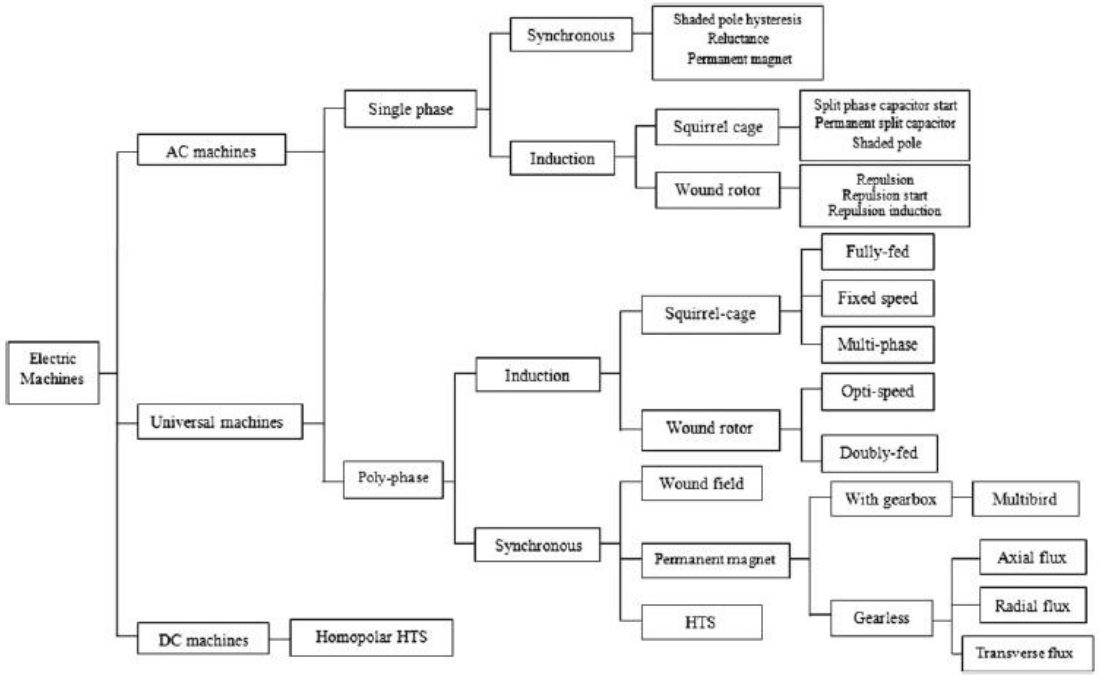
Generator

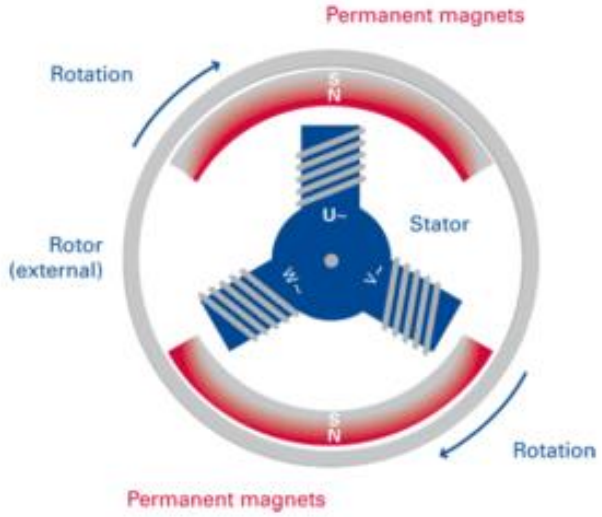
Power 'formatted' for grid



Other Designs

- This field is not completely stable, so new designs are still being investigated
- Some use a shell rotor and a core stator
- Africa sometimes uses smaller wind turbines because roads are too small to handle larger designs.





Vensys systems have the stator and rotor inverted to the standard design

- Economically, it is better to make big wind turbines
- This means really big generators
- Generators can be 30 tons, with 1/3 copper

Gearbox connected generator

Pic from RJWeng.com

Direct Drive (no gearbox))

Picture & Data from: Enercon & CopperAlliance.org, respectively

Physics of Generators

<u>Faraday's Law of Induction</u>

$$\Delta V_{ind} = -\frac{d\Phi_B}{dt}$$

<u>Gaus's Law for Magnetism</u>

$$\Phi_B = \int\int \vec{B} * \overrightarrow{dA}$$

$$\Phi_{\mathrm{B}} = BAcos\theta$$

$$\Delta P_{ind} = rF\omega sin(\omega t)$$

Magnetic moments due to spin

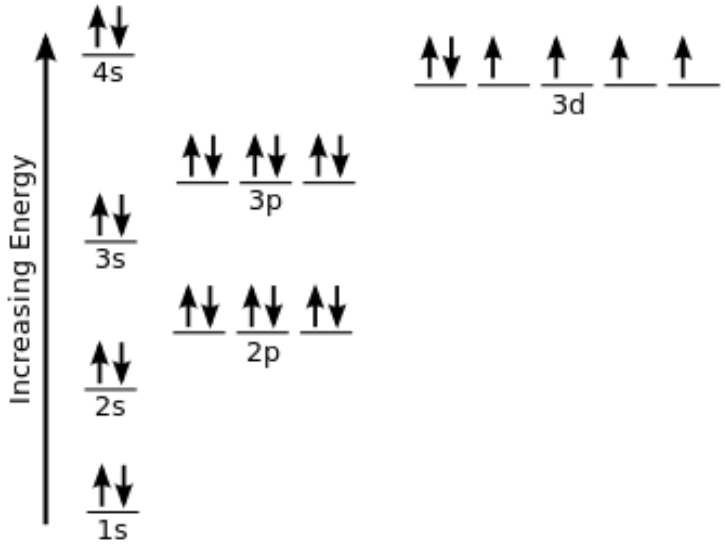
- Electrons spin can provide a dipole moment

$$\vec{\mu}_s = g_{g-factor}\frac{q}{2m}\vec{S}$$

Gg-factor= -2.00 (dimensionless)

- Spin up and spin down will cancel either other out.
- A rough estimate or dipole moment is to look at unparied electrons.
- However the exchange interaction (quantum switching of states) can negate ferromagnetism

Electronic states of Iron



Permanent Magnets

- We need an electric magnetic field to orient magnetic grains

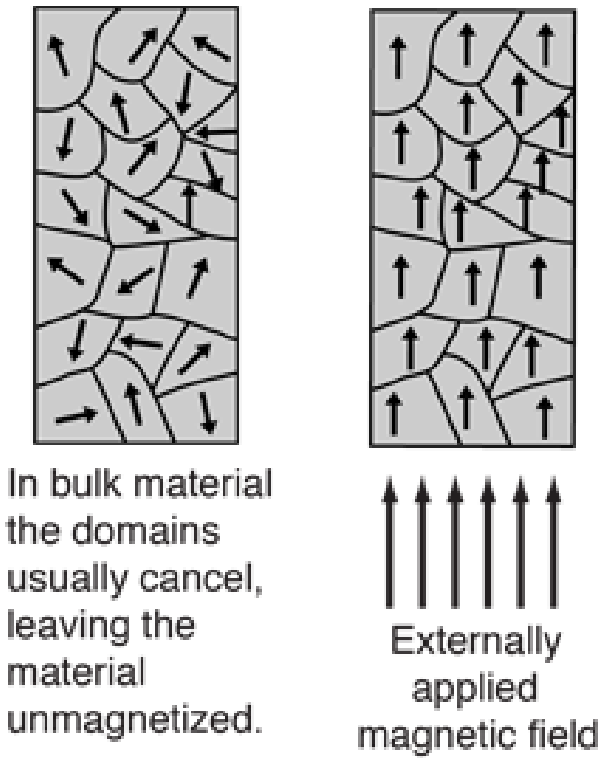


Image from http://hyperphysics.phy-astr.gsu.edu

- Once field is removed, materials stays magnetized
- Once magnetized, how do we distinguish between current induced magnetic field and magnetized field

$$B = \mu_{mag} H$$

Magnet field strength (A/m)

Magnet flux density (T)

Hysteris Charts

- We vary applied magnetization (H) and measure total magnetization (B).

$$B = H + \mu_0 M$$

$$M = \chi_m \frac{H}{\mu_0}$$

Measured via B-H graph and can vary with H

$$\mu_{mag} = (1 + \chi_m)\mu_0$$

$$B = \mu_{mag} H$$

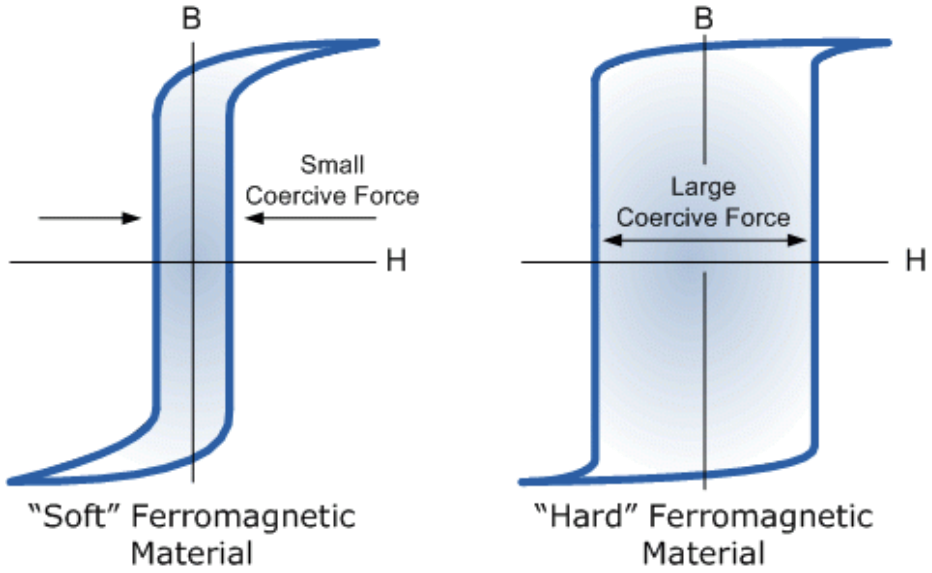


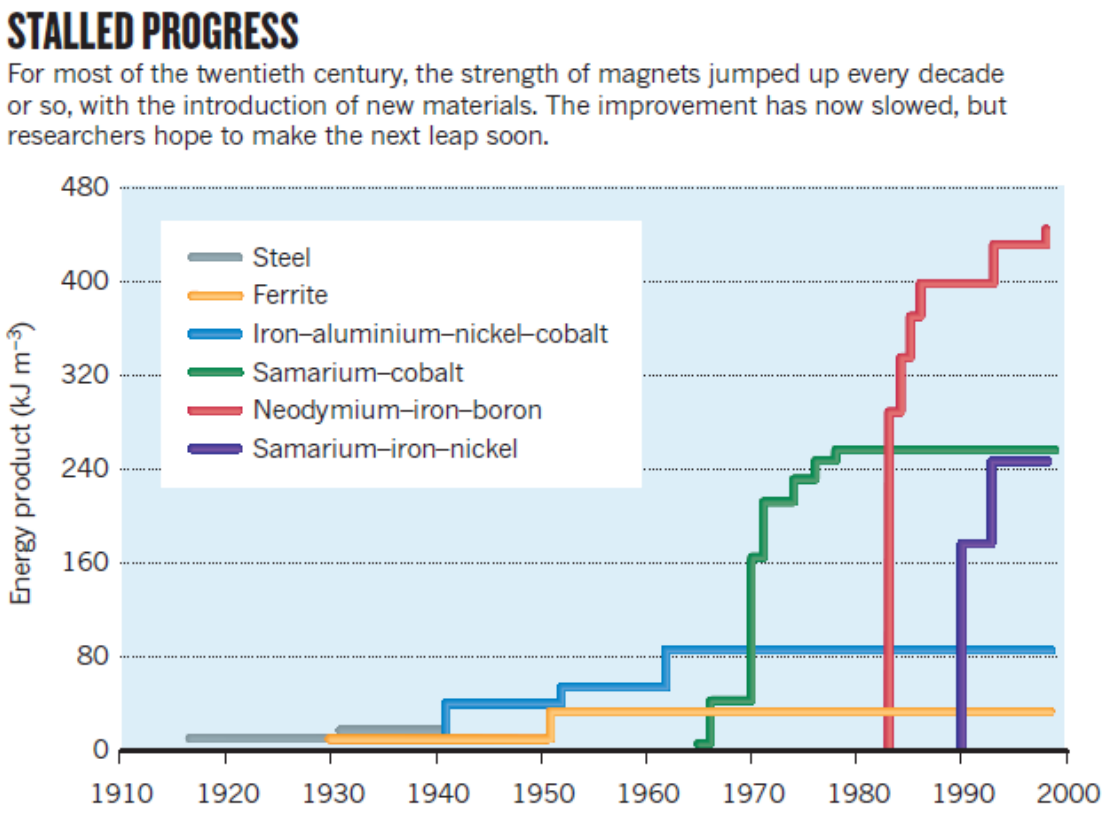
Image from https://www.electronics-tutorials.ws/

- Generators want hard magnets

H = Applied Magnetic Field
B = Total Magnetic Field
(Applied + Magnet)
M = Magnetization
χ_{mag} = Magnetic Susceptibility
μ_{mag} = permeability

Maximum B-H

- Rather than measure M or μ_{mag}, the product BH is typically used as a figure of merit.
- A higher BH means a better magnet.
- The unit of BH is kJ/m^3, but don't think of it as stored energy.
- Another unit sometimes used is the Mega Gauss-Orsted.
- 1 MGOe = 8 kJ/m^3

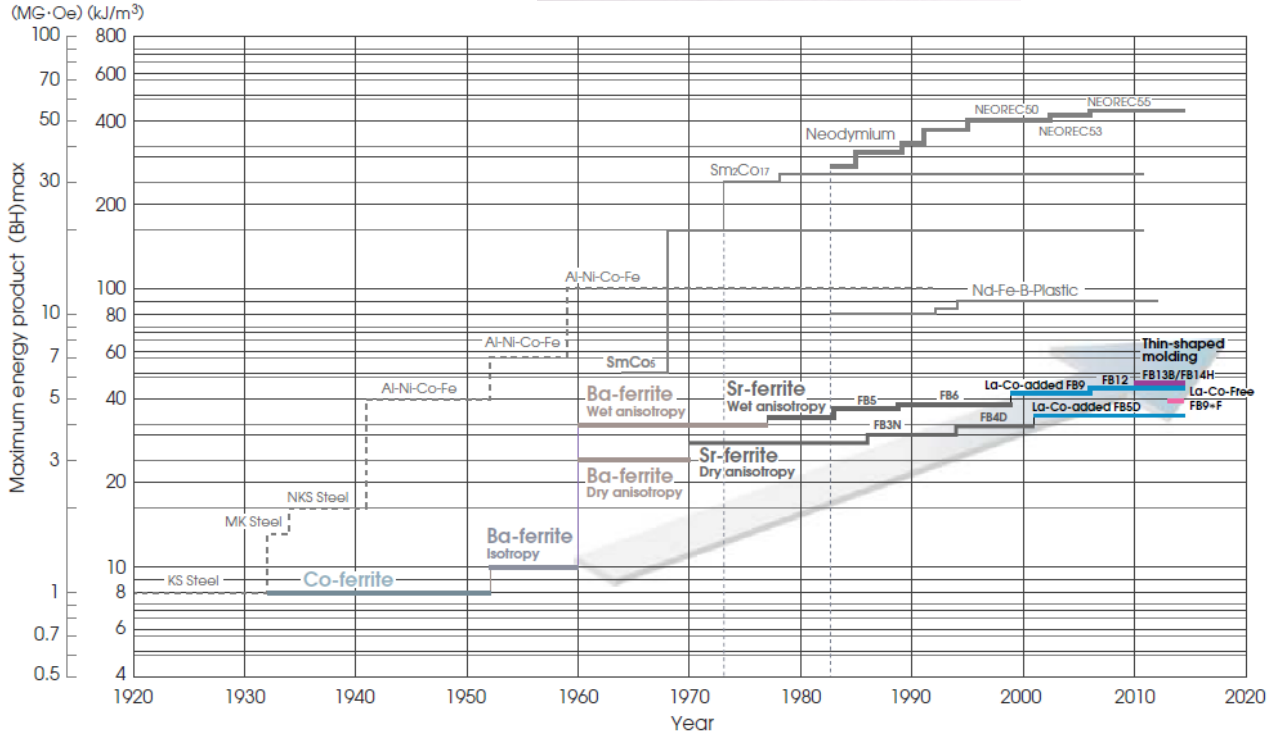


Ferrites (Fe_2O_3+ metal)

- Manganese and Nickel alloys give ‘soft’ magnets
- Strontium and Barium alloys give ‘hard magnets

- Since these are iron based, they are really cheap.

- In many situations, they are good enough



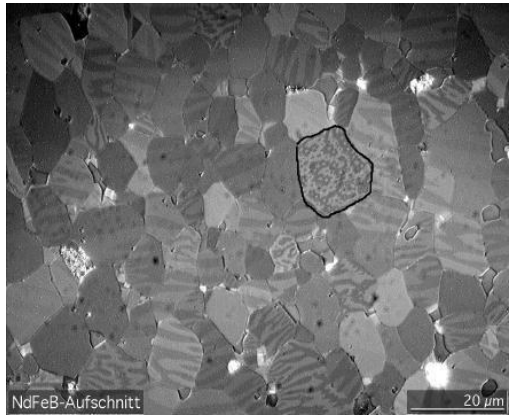
https://product.tdk.com/info/en/catalog/datasheets/magnet_fb_summary_en.pdf

$Nd_2Fe_{14}B$

- Developed in 1982 by General Motors and Susimoto (now Hitachi)
- High magnetocrystalline anisotropy (favors selective crystal growth)
- Nickel plating is usually used to prevent Corrosion.

Ni coated $Nd_2Fe_{14}B$

- 50 ktons/yr produced, 95% in China
- BH_{max} =516 KJ/m^3



Magnetic grains

Corroded $Nd_2Fe_{14}B$

$SmCo_5$ & Sm_2Co_{17}

- Developed in 1960's by Karl Strnat and Alden Ray
- High magnetocrystalline anisotropy (favors selective crystal growth)
- These are relatively corrosion resistant
- Advantages- Good at very high tempearture and very low temperature
- Disadvantages- Brittle, need high tempearture to synthesize
- BH_{max} = 112-264 kJ/m^3

$SmCO_5$ magnet

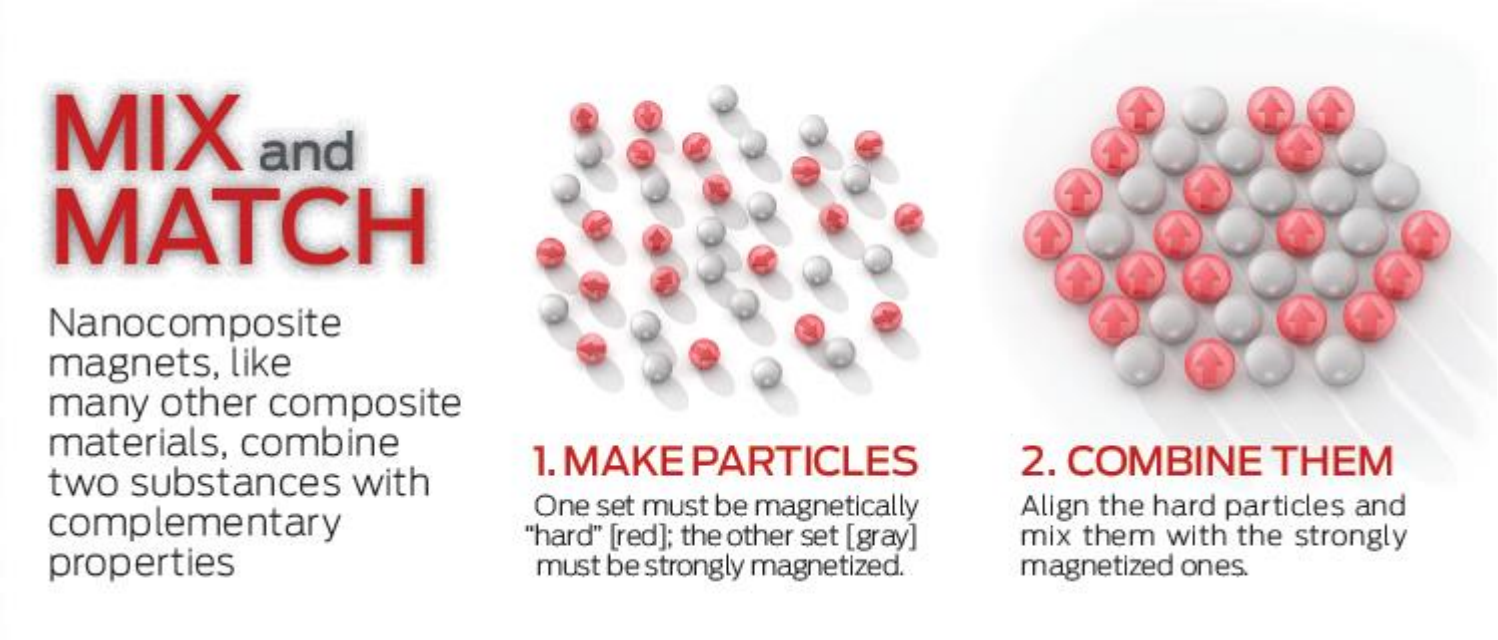
Comparison

- In general Neodynium is the best, unless you need to go to higher temperatures, then Samarium is.

Magnet	B_r (T)	H_{ci} (kA/m)	BH_{max} (kJ/m^3)	T_C (Curie Temperature)	
				(°C)	(°F)
$Nd_2Fe_{14}B$ (sintered)	1.0–1.4	750–2000	200–440	310–400	590–752
$Nd_2Fe_{14}B$ (bonded)	0.6–0.7	600–1200	60–100	310–400	590–752
$SmCo_5$ (sintered)	0.8–1.1	600–2000	120–200	720	1328
$Sm(Co, Fe, Cu, Zr)_7$ (sintered)	0.9–1.15	450–1300	150–240	800	1472
Alnico (sintered)	0.6–1.4	275	10–88	700–860	1292–1580
Sr-ferrite (sintered)	0.2–0.78	100–300	10–40	450	842

Magnetism at DTU Physics

- Magnetism and magnetic materials
  - Taught by: Catherine Frandsen
  - Spring F5B
- Catherine also researches nanoparticle magnets
- The basic stratergy is get a high ‘B’ material and a high ‘H’ material, and mix them to get a high BH material

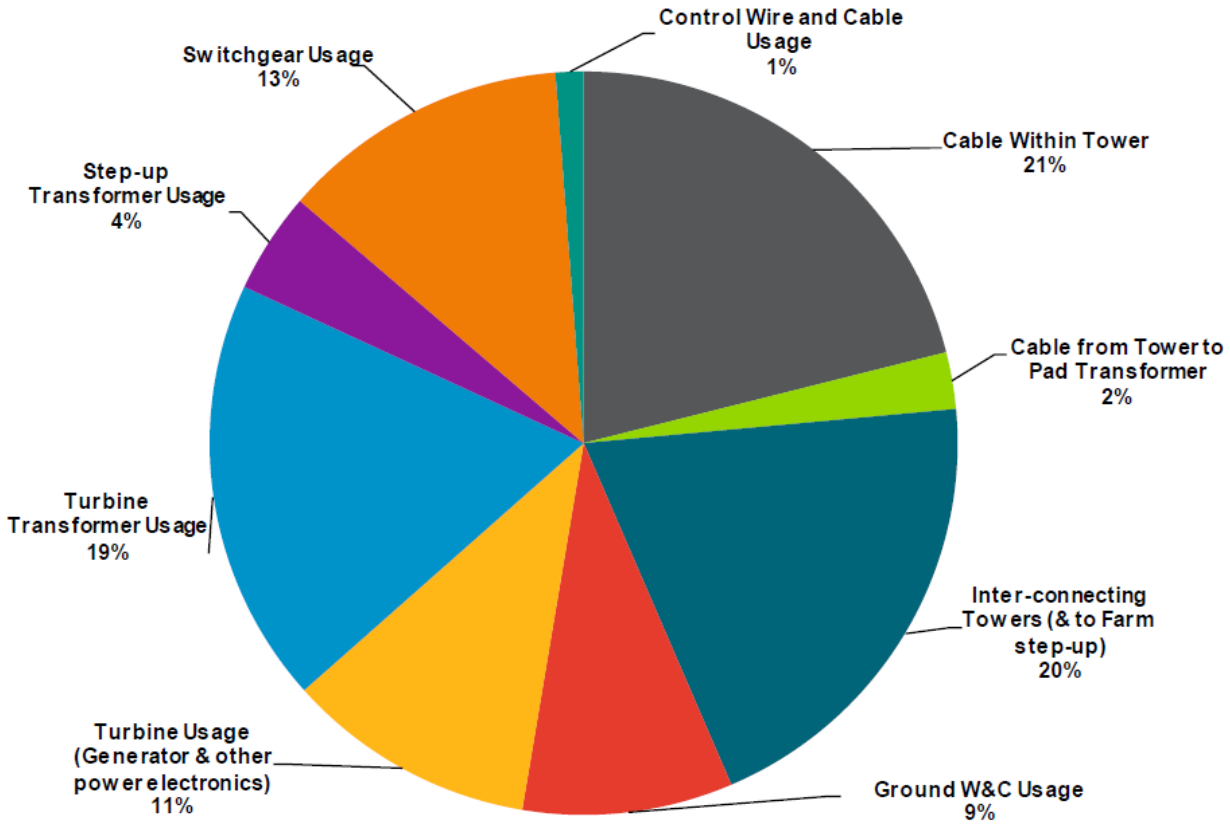


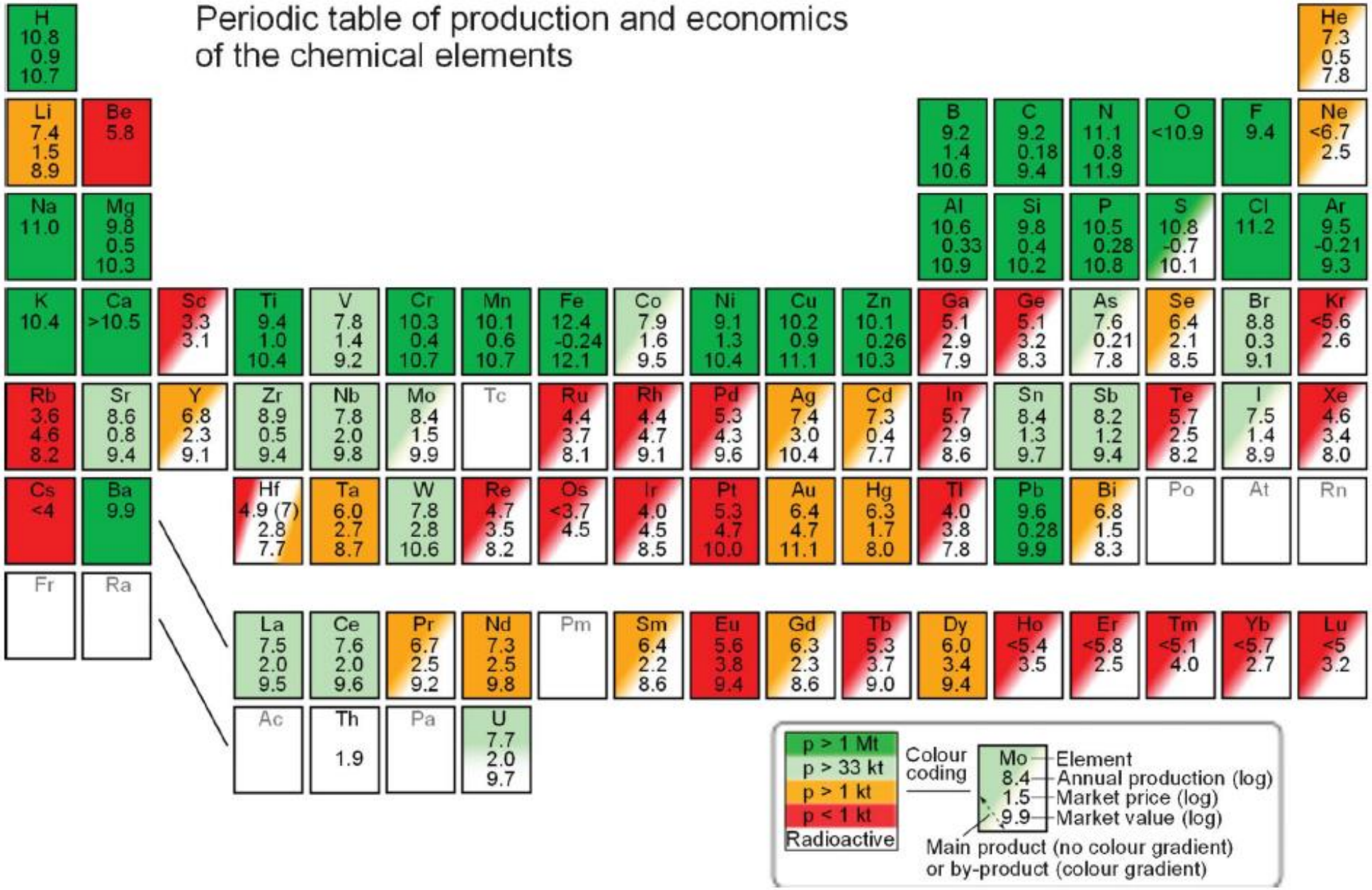
Materials

- Basically we are looking to produce 17-30 TW of energy from nothing.
- If a rare material is cheap now, will it still be cheap if we need 30 TW of it?
- Basically how much do we produce of a given material and is it enough?
- Lets look at wind turbines- what are the major components?

Copper in Wind Turbines

- Offshore Wind has 10 tons/MW of copper (ref Falconer, 2009)





Vesborg and Jramillo, RSC Advances, 2012,2, 7933–7947, Doi: 10.1039/c2ra20839c

Electro-Magnets

- By inducing a magnetic field via current, you reduce the need for permanent magnets

- Basically you just have 2 copper wires with different electrical currents/magnetic fields between them.
- 77% of wind turbine use electromagnets, but this will drop to 60% in next 10 years.
- Matching phases can be more complicated, but that is all done electronically.
- This is a little heavier and more expensive though.